Estimating Macroeconomic Uncertainty Using Information

Measures from SPF Density Forecasts

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Abstract

Information theory such as measures of entropy is applied into the framework of uncertainty decomposition. We apply generalized beta and triangular distributions to histograms from the SPF and obtain aggregate variance, average variance and disagreement, as well as aggregate entropy, average entropy and information measures. We find these two sets of measures are largely analogous to each other, except in cases where the underlying densities deviate significantly from normality. We find that aggregate uncertainty fell precipitously after 1992, whereas the disagreement between respondents was high before 1981. Our information measures suggest a more permanent reduction in overall uncertainty since the great moderation compared to other parallel studies. Using entropy measures, we find uncertainty affects the economy negatively. We find insignificant price puzzle in the inflation VAR framework. Information theory can be applied to capture the common shock and "news" in the density forecasts. We find that common shock accounts for the bulk of the individual uncertainty, it decreases as horizon falls and 'news' is countercyclical.

Key words: Density forecasts, uncertainty, Information measures, Jensen-Shannon information

1. Introduction

Macroeconomic policy makers are increasingly being interested not only in point forecasts, but also in the uncertainty surrounding them. Thus, estimating uncertainty has been an important challenge for forecasters because it not only reflects how confident forecasters are with their forecasts, but also has effects on the economy. Nicholas Bloom et al. explore several channels through which uncertainty affects the real economy (Bloom et al. (2007), Bloom (2009, 2014), Bloom et al. (2014)).

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Some prior researches estimate 'ex post uncertainty' by looking at forecast errors and its variants, whereas 'ex ante uncertainty' is of huge interests from economists, policy makers and financial market participants. Ex ante uncertainty could be estimated from survey data. As the survey datasets and survey formats evolve, the methodology of uncertainty estimation is also evolving. Before the utilization of density forecast surveys, variance of the point forecasts (disagreement) is a popular proxy for uncertainty. Researchers later looked into datasets that contain density forecasts (such as Zarnowitz and Lambros (1987), Lahiri, Teigland and Zaporowski (1988), Lahiri and Liu (2007), Boero, Smith and Wallis (2008, 2013)). This literature has attracted more and more attention in recent years. The surveys collect forecast data in the form of probabilities in bins, which can be drawn as histograms. The U.S. Survey of Professional Forecasters (SPF) is the longest standing survey of this sort. In the survey forecasters provide their subjective forecasts for the probabilities that a variable will fall into each of the predefined intervals. Similar survey datasets exist in Europe as the ECB Survey of Professional Forecasters, and in England as the Bank of England Survey of External Forecasters. Economists estimate uncertainty from the above density forecasts. Some popular proxies for 'uncertainty' include simple average of variances of individual density distributions, interquartile ranges of individual density variances (Abel et al. 2016), variances of pooled average distributions (Boero et al. 2013) and measures obtained from regressions (D'Amico et al. 2008). Using density forecasts instead of point forecasts, researchers find apparent distinction between disagreement and uncertainty in forecasting (Boero, Smith and Wallis (2008), D'Amico et al. (2008)). Thus estimating uncertainty from density forecasts is proved to be a superior approach than using point forecasts only, and disagreement may be a weak proxy for forecast uncertainty.

Most recently, a number of studies on the use of informatics in economic forecasting has evolved and provide insights into the information contained in the densities more generally (Kinal, Lahiri and Liu (2005), Rich and Tracy (2010), Shoja and Soofi (2017)). In this paper we utilize the information theoretic approach to develop alternative estimates for uncertainty. We in parallel

compare variance based measures and informatics based measures (including entropy and information divergence). We decompose the variance and the 'entropy' to analyze the aggregate uncertainty and its relationship with disagreement and other components.

Our procedure involves fitting a continuous distribution to the histogram data. Many previous studies that use the same dataset for various purposes have utilized raw histograms, assuming the probability mass is on the midpoint of the bin, see for instance D'Amico et al. (2008), Kenny et al. (2015). Others fitted normal distributions (Giordani & Soderlind 2003, Boero et al. 2013) or uniform distributions to the histograms to conduct the analysis (Abel et al. 2016, Boero et al. 2013). We adopt generalized beta distributions and triangular distributions in the estimation similar to that of Engelberg et al. (2006). It is shown that many of the probability forecasts in the SPF are not symmetric (Engelberg, Manski and Williams, 2006, Lahiri and Liu, 2009). We are the first to apply continuous distributions to histograms when utilizing information theory to estimate uncertainty. This enables us to carefully take care of all the histogram data and changing survey histogram bin sizes and bin numbers. With the generalized beta distribution as our main choice of fitting procedure, we propose an alternative, Jensen-Shannon information divergence, to the most popular Kullback-Leibler information divergence in our estimation. The Jensen-Shannon information measure conveys the same information but can fit into more histograms distributions than the Kullback-Leibler information measure. We further correct for different forecast horizons to show a continuous series of uncertainty over a long period.

The information measures obtained are then utilized in several application topics, including employing VAR to find effects of uncertainty on macro economy and estimating uncertainty shocks aka "news".

The paper is organized as following: in section 2 we briefly introduce the dataset and the key feature of including density forecasts. In section 3 we compare different measures of uncertainty by analyzing variances and verify previous findings that disagreement of point forecast should not be used as a proxy for uncertainty. In Section 4 we employ information theory to estimate

uncertainty and study the analogy between two approaches – the variance approach and the entropy approach. In Section 5 we correct for horizon effects and other minor issues to get the uncertainty over the long survey period and compare our measures to others'. In section 6 we evaluate the effects of uncertainty and monetary policy on macroeconomic variables in several VAR specifications. In Section 7 we apply information theory to find disagreement and common shock from the individual densities. In section 8 we apply information theory to individual data to estimate the 'news' from successive fixed target forecasts. Finally, Section 9 summarizes the main conclusions of the study.

2. The data

The U.S. Survey of Professional Forecasters (SPF) is a unique dataset that provides probability forecasts from a changing panel of professional forecasters. The forecasters are from financial institutions, universities, research institutions, consulting firms, forecasting firms, and manufacturers, etc.

SPF is currently maintained by the Federal Reserve Bank of Philadelphia. It was formerly conducted by the American Statistical Association (ASA) and the National Bureau of Economic Research (NBER). The survey was started in the fourth quarter of 1968 and was taken over by the Philadelphia Fed in the second quarter of 1990. A unique feature of the survey is its coverage of density forecasts, which is rare in North America. Similar siblings of the survey are Bank of England Survey of External Forecasters and European Central Bank (ECB)'s Survey of Professional Forecasters, which also feature some density forecasts from professional forecasters. SPF is conducted quarterly. The timing of the survey has been carefully managed to ensure forecasters have obtained the latest macroeconomic data and at the same time produce forecasts in a timely manner. The forecasters will have known the first estimate of a few quarterly macroeconomic variables for the prior quarter when he/she provides the forecast. For instance,

when providing forecasts in quarter 3, a forecaster has the information of the first estimate of the real GDP level by the Bureau of Economic Analysis for quarter 2.

The survey asks professional forecasters about their predictions of a bunch of variables, including real GDP, nominal GDP, price of GDP (GDP deflator), unemployment rate, etc. Forecasters provide their point forecasts for these variables for the current year and the next year, and for the current quarter and next three or four quarters. Besides the point forecasts, they also provide a density forecast, i.e. a probability forecast, for the output growth rate and price of output growth rate for the current and next year (though providing both forecasts is only started from the August 1981 survey). The definition of the 'output level' variable has changed in the survey several times. It was defined as nominal GNP before 1981 Q3 (third quarter of year 1981), and as real GNP from 1981 Q3 to 1991 Q4. After 1991 Q4, it is real GDP. The definition of the inflation variable 'Price of output' or 'Price of GDP' (PGDP) has changed several times as well. From 1968 Q4 to 1991 Q4, it stood for the implicit GNP deflator. From 1992 Q1 to 1995 Q4, it was implicit GDP deflator. From 1996 Q1 onward, it stands for GDP price index. The density forecasts for these two variables (output level and price of GDP) are probability forecasts of their annual growth rate in %, while the point forecasts are for absolute levels and have both the annual and quarterly values. For instance, a survey question of probability forecast of real GDP growth rate could look like this:

Sample survey question (partial) - Year 2013:

Part 1: Probability forecast. Please indicate what probabilities you would attach to the various possible percentage changes (annual-average over annual-average) this year (2013) in chain-weighted real GDP. The probabilities of these alternative forecasts should, of course, add up to 100, as indicated.

| Probability of indicated percent change in real (chain-weighted) GDP (%) |
|--|
| 2012-2013 |

| +3 percent or more | |
|----------------------|--|
| +2.0 to +2.9 percent | |
| +1.0 to +1.9 percent | |
| +0.0 to +0.9 percent | |
| -1.0 to -0.1 percent | |
| -2.0 to -1.1 percent | |
| Decline more than 2% | |
| TOTAL | |

Part 2: Point forecast. Please indicate the chain-weighted real GDP of year 2013. The real GDP is ___ billions dollars.

(Other questions for different targets and horizons are not shown here.)

In this paper, let's call each input of the first column of the above table a "bin" or "interval". To make it continuous, the numerical values of one sample bin "+1.0 to +1.9 percent" will be defined as [1, 2), and another sample bin "Decline more than 2%" will be defined as $(-\infty, -2)$. Let's call the bin $(-\infty, -2)$ or $[3, +\infty)$ a tailed bin (since they are at the tails of the range).

A forecast as following,

| Intervals | $(-\infty, -2)$ | [-2, -1) | [-1, 0) | [0, 1) | [1, 2) | [2, 3) | [3,+∞) |
|-----------|-----------------|----------|---------|--------|--------|--------|--------|
| Prob(%) | 0 | 0 | 70 | 30 | 0 | 0 | 0 |

would mean the forecaster estimates that there is a 70% chance that the growth rate of real GDP this year over last year would be between -1% and 0%, and a 30% chance that it falls into 0% to 1%.

As for the point forecast, it is in absolute levels but not growth rates. Therefore, to derive the point forecast for the growth rate of the variable this year over last year, we need the value of the variable for the year prior to the forecast year. This is possible by utilizing "The Real-Time Data Set for Macroeconomists" by the Philadelphia Fed, which tracks the historical values (vintage values) of macroeconomic variables. We use the value of the variable at the time the forecast was

made and thus do not need to worry about the various revisions of data such as first release, second release etc.

For two quarters (1985Q1 and 1986Q1) where the Fed is unclear about the horizon of the density forecasts, we compare forecasters' uncertainty measure from each survey to recent quarter results and can retrieve these data to identify the first density forecast as for horizon 0 and second forecasts as for horizon 4. In this extraordinary move we are able to derive a continuous data series for most variables of interest from the beginning of the survey.

3. Uncertainty, aggregate variance and its components

The earliest literature on uncertainty mostly estimates uncertainty by disagreement - variance of the point forecasts across forecasters. Suppose forecasters make point forecasts for the target variable y. Each forecaster i has a forecast $y_{i,t,h}$ with forecast horizon of h quarters for the target year t. For convenience we ignore the time and horizon subscripts, and write it as y_i for this part. In many previous works uncertainty is proxied disagreement, which is the variance of \mathcal{Y}_i ($\text{var}(\left\{y_i\mid_{i=1}^N\right\})$) across N forecasters. Various authors have proposed and tested that disagreement in many cases is not a good proxy for uncertainty. One approach is to look at how the true uncertainty can be decomposed and related to this proxy. Lahiri, Teigland and Zaporowski (1988) decompose the total variation into average uncertainty within respondents and disagreement measure between respondents. Boero, Smith and Wallis (2013) propose a similar identity. They both find that disagreement is only part of the aggregate uncertainty, is not perfectly correlated with the individual uncertainty or the aggregate uncertainty and hence is not a good proxy for uncertainty.

In this section we calculate aggregate variance, average variance and disagreement of means to show the decomposition and the trends in the components of aggregate uncertainty over the last few decades. In the process we fit generalized beta distributions or triangular distributions to the density forecast histograms. We also discuss the issue of the disparity between point forecasts and density forecast central tendencies.

For density forecasts with more than two intervals, we fit generalized beta distributions to them. When the forecaster attaches probabilities to only one or two intervals, we adopt a symmetric triangular distribution. The generalized beta distribution here is one derived from the beta distribution. Beta distribution has a base of [0, 1], while a generalized beta here has a base [l, r]. Thus a generalized beta distribution has four parameters, two parameters α and β which define the shape of the distribution, and l and r which define the base. The generalized beta distributions and triangular distributions are the preferred forms of distribution here for several reasons. First, many previous research do not assume a continuous distribution of the probability forecast. In those estimations the histograms are treated as discrete distributions, with the probability mass concentrated at the mid-point of each interval. This approach does not reflect the common feature of continuity of the true distributions. The discrete treatments also cause discontinuities in variance and information measures. Second, compared to the normal distribution, the generalized beta distribution is more flexible to accommodate the different shapes in the histograms. The histograms frequently display skewed shapes as well as different degrees of kurtosis. Generalized beta distributions are able to account for such shape properties for these cases. Further more, generalized beta distributions are truncated at both sides, while the normal distribution has positive densities till infinity, which is not true for most of the histograms and counterintuitive to the fact that the target variables have bounds in history.

Suppose forecaster i has a probability forecast in the form of a histogram p_i . p_i is a $k \times 1$ vector, corresponding to k intervals. $\sum_{j=1}^k p_i(j) = 1$. In the dataset some times $\sum_{j=1}^k p_i(j) \neq 1$ but

0.999 or 1.01, then they are rescaled to make the sum 1. We fit a continuous distribution f_i to p_i .

The mean and variance of f_i is μ_i and σ_i^2 . σ_i^2 is the uncertainty of individual i. If we take the average of σ_i^2 across all forecasters, we get the average uncertainty $\overline{\sigma_i^2}$. If we take the average of all the means we get the consensus forecast μ . The variance of the means μ_i , $\operatorname{var}(\mu_i|_{i=1}^N) = \frac{1}{N} \sum_{i=1}^N (\mu_i - \mu)^2$ is the disagreement about means. If we pool all individual distributions into a consensus distribution, we can obtain the aggregate variance as the variance of that aggregate distribution. Lahiri et al (1988) and Boero et al (2013) show the following identity $Aggregate\ Variance = disagreement\ on\ means + \overline{\sigma_i^2} \qquad (1)$

Multiplying both sides by N, we get another representation in the sense of total variation:

$$\sum_{i=1}^{N} \int f_i(x)(x-\mu)^2 dx = \sum_{i=1}^{N} (\mu_i - \mu)^2 + \sum_{i=1}^{N} \sigma_i^2$$
 (2)

In this identity, the left hand side is the total variation, which is the sum of all the variation across all forecasters from the consensus mean. On the right hand side we have the sum of squares of means deviations and sum of individual variances, which are N times of disagreement and average uncertainty respectively.

Equations (1) and (2) are equivalent. We look at equation (1) only. The equation shows a decomposition of variance/uncertainty. The aggregate variance is the sum of disagreement about means and average individual variance. Similarly, The aggregate uncertainty is the sum of disagreement about means and average individual uncertainty. Therefore, disagreement is only part of the aggregate uncertainty and may not be a good proxy for uncertainty if it does not correlate perfectly to the other two in the identity. Next we obtain these measures from SPF and look at the behavior of the three measures.

To get the aggregate variance, we have two approaches. In approach 1, we get the consensus histogram $\frac{1}{N}\sum_{i=1}^N p_i$ first, and then fit a generalized beta distribution f_a to the histogram, and obtain the variance of f_a , $\sigma_{f_a}^2$. In approach 2, we utilize the individual generalized beta and triangular distributions that we have fitted to the individual histograms. We take the average of them to get the consensus distribution $f_c = \frac{1}{N}\sum_{i=1}^N f_i$, and compute the variance of f_c , $\sigma_{f_c}^2$. Since the individual distributions are not linear, $f_a \neq f_c$. f_a is a smooth generalized beta distribution, while f_c is an unsmooth distribution as the average of many generalized beta and triangular distributions.

Figure 1 shows the series of aggregate variance, disagreement on means and average individual uncertainty for output forecasts at horizon 4 (i.e. made in the first quarter). Upper panel is computed by the first approach, and lower panel is computed using the second approach. In both panels, the red and black curves are the same, representing disagreement on means and average individual variance. The blue curve stands for aggregate variance, computed by different approaches as explained above. Since in approach 2 the identity (1) will be strictly held while in approach 1 it does not due to the smoothing, we adopt approach 2 in computing the aggregate variance.

Figure 2 shows the series of aggregate variance, disagreement on means and average individual variance, for forecast horizons of 1 to 8 quarters, for output and inflation forecasts, respectively. All computations are using approach 2 steps. In each graph the black vertical lines split the graph into three periods – before 1981Q3, between 1981Q3 and 1991Q4, and after 1991Q4. For output forecasts, the definition has changed. For output and inflation forecasts, the number of predefined intervals and the interval length both changed. The interval length between 1981Q3 and 1991Q4 was 2, while in other periods it was 1.

These 16 graphs show how the aggregate variance and its two components evolve as time and forecast horizon changes.

For output forecasts, uncertainty before 1981 was the highest. This is partially due to the fact that the definition of the target variable was nominal GNP, and the U.S. experienced volatile inflation in the 1970s and early 1980s such as the oil crises, so the large uncertainty in forecasting nominal GNP is expected. The recessions in the period also explain the volatility.

For the period between 1981Q3 and 1991Q4 output growth uncertainty was high as well, but it is partially caused by the length of interval being 2 rather than 1. Smaller number of bins and longer bins reduce the accuracy and the embedded information of the forecasts, and increases variances obtained.

When we look at the two components of aggregate variance, we find that before 1981 disagreement accounts for most of the aggregate uncertainty/variance, while after 1981, it plays a less prominent role. Before 1981, every hike of aggregate variance is almost the result of a hike in the disagreement about means in similar magnitudes, while between 1981 and 1991, most hikes in aggregate variance are resulted by higher average variances. It also seems that the average variance is much less volatile than the disagreement over the full sample period. Average uncertainty only picks up a little during 1980s. We propose that it is partially due to the fact the length of interval being 2 rather than 1 and will address this issue in a later section.

Generally, all three series tend to decrease as forecast horizon shortens. This is as expected since as more information ('news') comes in, forecasters become more certain of the current year's economy.

For output forecasts, the recent two decades have seen a persistent low level of disagreement. But disagreement soars quickly and excels average uncertainty in recession quarters, such as in 2008 and 2009. It can be interpreted that during economic downturns, forecasters still hold their uncertainty, but they disagreed more about the mean forecast.

For inflation forecasts (GDP deflator), similar patterns exist. The period before 1981 was the most volatile, and disagreement accounted for a larger portion of the aggregate variance, or say between variation accounted for a larger portion of the total variation. For the period between 1981Q3 and 1991Q4 inflation uncertainty was high as well, but disagreement played a less important role except in two quarters. The period after 1991Q4 is the most peaceful and has seen a steady decrease of overall uncertainty.

For some quarters before 1981Q3 we do not have data for the longer horizon forecasts (6~8 quarters) and 1 quarter forecasts, thus we cannot see the highest hikes in these volatile quarters in the longer horizon graph.

Given the relations of the three series, we can conclude disagreement on means should not be used as a proxy for uncertainty, especially in the recent two decades. Hence, the variance of the point forecasts is not a good proxy for uncertainty. Furthermore, the variance of the point forecasts has another flaw. Lahiri and Liu (2009), Engelberg et al. (2006), Clements (2012) etc. find the point forecasts deviate from the density central tendencies and forecasters tend to provide more favorable point forecasts than their corresponding density central tendencies. Elliot et al. (2005) propose theoretical foundation for the rationale and find asymmetric loss functions in other datasets as well. We previously (Lahiri and Wang 2014) find forecasters systematically make higher real GDP point forecasts than the central tendency of underlying density forecasts in good times and lower point forecasts than the density median in bad times. They provide higher inflation point forecasts than density medians when the level of inflation is high and lower point forecasts when it is low. Thus the disagreement about point forecasts is even less informative than disagreement about density forecast means. Given these reasons, we should not use variance of point forecasts as a proxy for uncertainty.

4. Uncertainty and information measures

Recently the information theories are incorporated into the forecast analysis. Soofi and Retzer (2002) give a few different information measures and estimation methodologies. Rich and Tracy (2010) first apply information theory into estimation of uncertainty and calculate entropy of forecasts. Shoja and Soofi (2017) make analogy between information measures and variance decomposition. In this paper we elaborate on their results and provide an alternative information measure which can be applied to the generalized beta distributions that we got in the fitting procedures. The same analogy can be made when comparing it to the variance decomposition in Section 3.

In information theory, entropy is a proxy for all the information contained in a distribution.

Shannon Entropy is defined as

$$H(f) = -\int f(y)\log(f(y)) dy$$

Shannon Entropy is to measure how close a distribution is to an infinitely dispersed distribution. The higher the entropy, the less information it contains in the distribution, the higher level of uncertainty. We can make an analogy of entropy to variation. Generally, the larger the variation, the less information, and the higher the uncertainty. Thus the entropy is an analog of variance. But entropy, by its definition, contains more information than the variance statistic, because different distributions may have the same variance, but their shapes may be different and thus have different entropies. López-Pérez (2015) proves that entropy satisfies several properties as a "coherent risk measure" whereas moment based measures such as variance does not. Entropy is superior to variance especially in cases when the underlying distribution is not unimodal (and potentially discontinuous).

The divergence between two distributions is related to the entropy. A popular divergence measure is the Kullback-Leibler information measure (KL) or Kullback-Leibler divergence. For two distributions (pdf) f(x) and g(x), the KL information measure is defined as

$$KL(f(x),g(x)) = \int f(x) \log \frac{f(x)}{g(x)} dx$$

The KL information measure is only valid when $g(x) \neq 0$. This has a lot of limitations to the beta shaped distributions. Besides this, the KL information measure is not symmetric, i.e. $KL[f(x),g(x)] \neq KL[g(x),f(x)]$, if certain conditions do not hold.

We propose another information measure – the Jensen-Shannon information measure (JS). For two distributions f(x) and g(x), the JS information measure is defined as $JS(f(x),g(x)) = \frac{1}{2} (\int f(x) \log \frac{f(x)}{m(x)} dx + \int g(x) \log \frac{g(x)}{m(x)} dx)$

where

$$m(x) = \frac{1}{2}(f(x) + g(x))$$

The JS information measure is symmetric and is suitable for the cases when the densities of f(x) and g(x) are on different intervals.

The information measure could be viewed as an analogy to disagreement but it contains more information than the 'disagreement about means' measure which is only a variance of the means vector. It captures the differences in all aspects of the shapes of the distributions rather than the difference of the mean only.

Given the analogy of the above measures, we can make an analogy between variance decomposition and entropy decomposition.

Entropy of aggregate distribution = Average individual entropy + Information measure (3)
We first obtain the estimate of 'Entropy of aggregate distribution' in two steps similar to approach 2 in Section 3. We utilize the individual generalized beta and triangular distributions that we have fitted to the individual histograms. We take the average of them to get the consensus

distribution
$$f_c = \frac{1}{N} \sum_{i=1}^{N} f_i$$
, and compute the entropy of f_c .

The second term, 'average individual entropy' is obtained by taking the average of entropies of f_i across forecasters. Figure 6 shows the high correlation between individual entropies and variances. It supports the idea that entropy and variance both measure the uncertainty of a forecast. When we take the natural logarithm of variances, the relationship becomes linear and very significant. It is clear that in the second graph in Figure 6, most points fall on a frontier which is a straight line. We check the properties of the points that fall apart from this frontier and find that these deviated points represent distributions that are significantly more skewed than those on the frontier. We propose this may be further used as a way to check the normal property of a distribution, but using individual data in SPF we did not find a significant (negative) correlation of the two when we regress the deviation of the points from the frontier line on the normality of the distribution. The deviation is mostly correlated with excess kurtosis and skewness.

The third term of equation (3), 'information measure', captures all the disagreement among f_i .

With N forecasters, we can compute the JS information measure for $\frac{N(N-1)}{2}$ pairs of distributions. But this approach is computational inconvenient. The second approach is to compute the JS information measure for N pairs of distributions, i.e.

$$\frac{1}{N}\sum_{i=1}^{N}JS(f_{i},f_{c})$$

However, this measure has relatively small values in all quarters and makes almost no contribution to the variations of aggregate entropy. Besides, this information measure does not make equation (3) held. While $\frac{1}{N} \sum_{i=1}^{N} JS(f_i, f_c)$ seems to be a natural analogy of disagreement

$$\frac{1}{N}\sum_{i=1}^{N}(\mu_i-\mu)^2$$
, it is not satisfying here and we propose another form, the generalized Jensen

Shannon information measure. A generalized Jensen-Shannon information measure for more than two distributions is in this form:

$$JS(f_1, f_2, \dots f_N) = H(f_c) - \frac{1}{N} \sum_{i=1}^{N} H(f_i)$$
(4)

where H(.) is Shannon entropy.

This form of JS information measure is closely related to Kullback-Leibler information measure:

$$JS(f_1, f_2, ..., f_N) = \frac{1}{N} \sum_{i=1}^{N} KL(f_i, f_c)$$

In this context, we take the difference of $H(f_c)$ and average entropy $\frac{1}{N}\sum_{i=1}^N H(f_i)$ to get the information measure. We display a three-series graph to show the entropy decomposition. In Figures 3, three series, the aggregate entropy, average entropy, and information measure are displayed by forecast horizon. Figure 3.A. is for output forecasts and Figure 3.B. is for inflation forecasts. In these graphs, aggregate entropy is $H(f_c)$, average entropy is $\frac{1}{N}\sum_{i=1}^N H(f_i)$, and information measure is obtained from equation (4).

We find that output and inflation forecasts have similar uncertainty patterns across all forecast horizons, and in the three periods – pre-1981Q3, 1981Q3-1991Q4, and 1992 onward. Pre-1981Q3 was characterized by the high overall uncertainty (high aggregate entropy), and high disagreement (high information measure). 1981Q3-1991Q4 was characterized by the big contribution of average entropy to the aggregate entropy. Post-1992 era sees smaller aggregate entropy. As horizon decreases, uncertainty falls, but the fall from horizon 8 to 7 until horizon 3 is quite small. Only from horizon 3 to 2 and 2 to 1, the uncertainty falls sharply. It indicates the high importance of news in the second half of a year and the sharply increased confidence when the forecast horizon is very short. It seems news or data of the second and third quarter of each calendar year has important indications for forecasters and they rely on these real time data

heavily to make forecasts. We can also guess that when forecasters observe turbulence in growth rates, they will not adjust their uncertainty very much for next-year forecasts, but only will take advantage of the new data to adjust the current-year forecast where the new data is part of the target variable, or say, they will only adjust their uncertainty measure after they observe partial realization of the target variable.

Entropy and information measure in Figure 3 are analogy to aggregate variance and disagreement on means in Figure 2. We compare Figure 3 to Figure 2, respectively. In Figure 2, from longer horizons to shorter ones, and from 1970s to the 2000s, uncertainty falls significantly. It is also reflected in Figures 3, but the fall is not that large. In recession era, uncertainty expressed as variance spikes sharply, while uncertainty measured as entropy only increases mildly. The same for disagreement when comparing variance of means to information measure. Meanwhile, disagreement on means may contribute to more than 80% of the total variation, but disagreement on distributions (information measure) at most contribute to about half of the aggregate entropy. Uncertainty in the period of 1981 to 1991 was high, and we have to point it out that in this period the survey design had a bin length of 2. To see the effect of this larger bin length on outcomes, we re-calculate the uncertainties of output forecast in other periods assuming they have a two-point bin length as well. We combine probabilities in adjacent bins and re-fit a continuous distribution to the new histogram with 2-point bins, and then calculate all relevant measures. The result is shown in Figure 7 and 8.

In Figure 7 we plot the aggregate entropy/variance with the 1-point bin length and 2-point bin-length, for 2-quarter-horizon output forecasts. Uncertainty when using a 2-point bin length after 1991 is frequently higher than that using a 1-point bin. But in the 1970s, a change to a 2-point bin length did not cause the uncertainty to be much higher. In Figure 8, we plot the uncertainty calculated from the two approaches against each other and find that when uncertainty is low, the structure of bins (bin length) will have a larger effect on the outcome and possibly causes the entropy measure to be higher (plots farther from 45 degree line). A possible explanation is that

when entropy is high, the densities span to many intervals and after a change of the bin length from 1 point to 2 points, there are still enough bins with positive probabilities to provide enough information for a good shaped continuous distribution fitting. However if entropy is low, after the change, the number of bins may shrink to 1 or 2, which strongly affect the estimate of entropy.

Thus we run two regressions to correct the bin length for aggregate entropy above 1.5 and below 1.5, respectively. Aggregate entropies in 1981-1991 are corrected to be a little lower using the simple regressions. We also run a regression to correct the bin length for average entropy.

After the correction for bin length for entropies, we apply equation (4) to recalculate the information measure. Due to the larger downward correction for average entropy, the information measure in the 1-point bin length case is larger than that in 2-point bin length case.

With the new estimates of entropy measures, we re-plot the three series of entropy measures for current year forecasts in Figure 5. For the period of 1981 to 1991, both aggregate entropy and average entropy fall and the aggregate entropy is not as high as the 1970s any more.

We want to point out that the continuous distribution fitting procedure is preferred by the results here, which is an improvement from Shoja and Soofi (2017). In Figure 9 we can see that fitting with continuous distributions is more reasonable than treating it as discrete data: we plot the two measures of individual entropy against each other we can see there is a clear disconnection (two parallel lines) which reflects some discontinuities in the computation in the discrete case.

5. Time series of uncertainty measures

After correcting for bin length, we correct all estimates for horizons. The median value of entropy measures for each horizon is subtracted and then the average of horizon 2 is added to make all observations horizon-2 equivalent. Thus we can have a complete view of the uncertainty in the last few decades. In Figure 10, the aggregate entropy for output forecasts and inflation forecasts are shown.

For output forecasts, uncertainty fell from the peak to a trough from 1980 to 1995. This fall of uncertainty seems to be a permanent fall. It does not reach the previous peak even in the 2009 financial crisis.

In the fourth quarter of 1988 and 1991 there were two high spikes, but that is partially due to the fact that uncertainty is generally very low in the fourth quarter, thus a modestly high value will be corrected to a very high value when we take out the median. Except these two quarters, the uncertainty is in a persistent down way in that decade.

Uncertainty picked up in the 2008/09 financial crisis, and peaked in the end of 2008, coincident with the dramatic events of that quarter - Lehman Brother's bankruptcy, the government's bailout, etc. A concerning trend now to us is that uncertainty of output forecasts is in an upward trend again in the last two years, suggesting modest risks in our economy.

For inflation forecasts, uncertainty remained high from 1970s until early 1990s. Then it went into a downward passageway. Similar to the output forecast uncertainty, the fall of inflation uncertainty in the 1990s seems to be permanent, and the new peak in the 2008 financial crisis is quite low compared to 1970/80 levels. Unlike output forecasts, inflation forecast uncertainty enters a downward trend from 2010 until present, suggesting a persistently low inflation environment in the U.S. economy. The aggregate variance for output and inflation forecasts are also shown here. They behave similarly to entropy measures for long run trends. But they also show a few different features, including higher volatility of variances and a higher peak for output forecast in the 2008 peak.

A further comparison of uncertainty as aggregate entropy to other uncertainty measures in the literature (particularly, Jurado et al 2015 and Baker et al. 2016) is shown in Figure 11. Jurado et al's uncertainty measure is macroeconomic uncertainty, and Baker, Bloom and Davis' is macroeconomic policy uncertainty. Like entropy, all measures spiked during recessions and reached troughs in the great moderation period of 1990s and before the dawn of the financial crisis in 2005. However the magnitudes and persistency of the spikes are quite different. Our

entropy measures suggest a permanent fall in uncertainty since the 1980s, while Jurado et al's and Baker et al's both saw historically high peaks in the 2008 crisis. Baker et al's high policy uncertainty is also persistent in the last few years even when the economy has recovered quite a lot. It can be explained by the extraordinary policy environment after the financial crisis, like the unprecedented Quantitative Easing policies. Our entropy measures suggest that, even in the significant events like the financial crisis and QE, forecasters remain relatively confident with their forecasts for output and inflation.

6. Impacts of uncertainty on macroeconomic variables

There has been extensive research on impacts of uncertainty on macroeconomic variables. We first explore impacts of output forecast uncertainty on the macro economy. Nicholas Bloom et al. explore several channels through which uncertainty affects the real economy (Bloom et al. (2007), Bloom (2009, 2014), Bloom et al. (2014)). Following the literature and particularly Baker et al. (2016) and Jurado et al. (2015) we use vector autoregression (VAR) models with Cholesky decomposed shocks to study the effects. We include four lags due to its minimized AIC. We consider several specifications. In our base model, we run the VAR of the following six quarterly variables: log of real GDP, log of nonfarm payroll, log of private domestic investment, federal funds rate, log of S&P 500 index and aggregate entropy of output forecasts. We draw the impulse responses of all variables to a one standard deviation change of the entropy, and the Cholesky ordering is the same as above. We find that an increase of output uncertainty has negative effects on GDP, employment, private domestic investment, interest rate and stock index, as shown in Figure 12A. The effects are not significant for most periods. Uncertainty itself converges back quickly after the initial shock. We also find that change of the ordering of variables does not alter the results. Specifications with fewer variables produce similar results that output uncertainty has negative effects on these variables. The negative relationship between uncertainty and economic activities is widely reported by previous research, such as Bloom (2009, 2014), Bloom et al. (2014), Jurado et al. (2015) and so on.

How monetary policy affects macro economy and uncertainty is of huge interests by researchers. Here we impose a one standard deviation shock to the federal funds rate in this specification and find that, uncertainty increases and other macroeconomic variables are negatively affected, shown in Figure 12B, which is expected as the depressing effects of tightening monetary policy.

Next we consider the impacts of inflation uncertainty over other macroeconomic variables. Most existing research do not have inflation uncertainty in their framework. In the field of inflation research, much attention is paid to the relationship between the trio of inflation, output and monetary policy. These researches where VAR is employed find the famous "price puzzle" -response of inflation to tightening monetary policy (a rise in interest rate) is positive, which is counterintuitive. Many have proposed possible explanations and modifications to the baseline VAR framework. Some examples include adding commodity prices into the VAR (Sims 1992), adding variables reflecting inflation expectations and adding restrictions to coefficients to reflect lags of policy effects in VAR. Some believe the reason of the price puzzle is that Federal Reserve raises interest rate to counter inflationary pressures, and either the policy is not strong enough or has lags in making an effect, both leading to the observed positive correlation of inflation and interest rate. Balke and Emery (1994) find that adding interest rate spread between 10-year and 3month treasury notes solves the price puzzle. It confirms the view of the inadequate efforts of the Fed to counter inflation pressures from negative supply shocks. Brissimis and Magginas (2006) add two forward-looking variables into the baseline VAR and solve the price puzzle. Estrella (2015) imposes one restriction to the coefficient of VAR to control for the policy lag and solves the price puzzle.

We run VAR models with four variables, i.e. log of GDP deflator, log of real GDP, federal funds rate and aggregate entropy of inflation forecasts. We consider two scenarios, a shock in inflation uncertainty (Figure 13 panels A and C), and a shock in monetary policy (Figure 13 panels B and

D). We run the model with two samples, one is from 1968Q4 to 2015Q4, and the other is from 1981Q3 to 2015Q4. We run the shorter period sample due to several reasons, including the limited availabilities of data before 1981 (thus we use both current year and next year forecasts to derive the entropy series), and the fact that inflation data in the 1970s and early 1980s demonstrate unusual volatility due to the two oil crises and the VARs become very sensitive to a few outliers. We find that an increase of the federal funds rate leads to lower output and higher inflation uncertainty, consistent with most research. A shock in inflation uncertainty leads to lower interest rates and lower output. This is consistent with Cheong et al. (2010) and Sauer et al. (1995). We also notice that, when imposing a shock to interest rate, inflation uncertainty goes up whereas when imposing a shock to inflation uncertainty, interest rate goes down, making the bilateral correlation with different signs in these two scenarios. Our results can help explain why findings of the correlation between inflation uncertainty and inflation are mixed.

In Figure 13, the price puzzle exists for the full sample of 1968Q4 to 2015Q4, but is not apparent in the sample of 1981Q3 to 2015Q4. We suggest that the proposition (that inadequate counter inflation efforts by the Fed in supply shocks could explain price puzzle) of Balke and Emery (1994) is plausible here since the U.S economy experienced significant negative supply shocks in the 1970s. The inexistent price puzzle in the recent sample reflects less severe supply shocks recently. We also tried adding log of producer price index to the VAR framework but it does not change the direction of the impacts of other variables.

7. Common shocks in density forecasts

Lahiri and Sheng (2010) propose a framework to find the common shock of macroeconomic variables, a link between individual uncertainty and the common shock that others seldom perceive. We apply the framework to the entropy data here.

Suppose a forecaster's forecast has distribution (pdf) f_i and the unknown consensus forecast pdf is f_{λ} . Following Lahiri et al (2010), we assume that when the forecaster makes that forecast, he is incorporating his own forecast to the consensus forecast. The consensus forecast comes from all the common information, and contains the common shock from the forecast date to the target date. We assume the forecaster provides a forecast $f_i = \frac{1}{2}(f_{i\varepsilon} + f_{\lambda})$, where $f_{i\varepsilon}$ is the adjustment he incorporates into the consensus forecast. From the properties of entropy, we have

$$H(f_i) = \frac{1}{2} [H(f_{i\varepsilon}) + H(f_{\lambda})] + Info(f_{i\varepsilon}, f_{\lambda}).$$

If we assume:

(a)
$$\frac{1}{N} \sum_{i=1}^{N} H(f_{i\varepsilon}) = H(f_{\lambda})$$
, i.e. the average adjustment is close to zero;

(b)
$$Info(f_i, f_{\lambda}) \approx Info(f_i, f_{\epsilon})$$

then we will have

$$\frac{1}{N} \sum_{i=1}^{N} H(f_i) = H(f_{\lambda}) + \frac{1}{N} \sum_{i=1}^{N} Info(f_i, f_c)$$

$$H(f_{\lambda}) = \frac{1}{N} \sum_{i=1}^{N} H(f_{i}) - \frac{1}{N} \sum_{i=1}^{N} Info(f_{i}, f_{c})$$

i.e.,

Common shock = average entropy –average information measure

Where average information measure =
$$\frac{1}{N} \sum_{i=1}^{N} JS(f_i, f_c)$$

With the entropy and information measures obtained in previous sections we are able to get the values of common shock for each quarter. It is shown in Figure 14.

We find that common shock displays strong horizon effects, which is as expected since common shock is the cumulative shocks from the forecast date to the target date, thus the longer the

forecast horizon, the larger the common shock. However the common shock almost contribute to more than 90% of individual entropy and nearly 100% since 1980. There are two explanations. First, information measure in the 1970s was the highest, causing the calculated common shock to be smaller since we define common shock as average entropy minus information measure. Second, we make two assumptions (a) and (b) in the above setup, and these assumptions may unexpectedly imply forecasts only make small adjustments to the consensus forecast. Besides the possible explanations here, we can conclude from the data that, under the assumptions in this section, on average forecasters make their forecasts based on the common shock and only incorporates a disagreement term (information measure) at a small scale.

8. Information measure and news

How "news" affects economy is frequently looked upon by economists. Forecast revisions are the most popular proxy for estimation of "news". Whereas we propose a more comprehensive measure derived from informatics. The Jensen-Shannon information measure could be applied to individual observations to estimate the 'news'. In the SPF surveys forecasters make fixed-target probability forecasts, making it possible to check how forecasters update their forecasts quarter to quarter. If a forecaster updates his forecasts for the same target variable from $f_{i,t,h}$ to $f_{i,t,h-1}$, we can obtain the disagreement/divergence of these distributions, which is $JS(f_{i,t,h}, f_{i,t,h-1})$. This measure could be viewed as 'news'. It is new information that makes the forecaster to update his forecast.

People are not only interested in the value of news, but also the variance of news. We can interpret the variance of news as how differently news affects different forecasters. We develop a measure to capture the variance of news below. For convenience we denote the two distributions as f(x) and g(x).

Variance of news =
$$\frac{1}{2} \left(\int f(x) (\log \frac{f(x)}{m(x)} - \mu_1)^2 dx + \int g(x) (\log \frac{g(x)}{m(x)} - \mu_2)^2 dx \right)$$

Where
$$m(x) = \frac{1}{2}(f(x) + g(x))$$
, $\mu_1 = \int f(x) \log \frac{f(x)}{m(x)} dx$ and $\mu_2 = \int g(x) \log \frac{g(x)}{m(x)} dx$

Figure 15 shows the values of news and the variance of news.

We find that forecasters were faced with more news in the 1970s. In the recent decades, only a few quarters see higher values of news. 'News' shows a slightly countercyclical property, that it hikes in the recessions of early 1980s, 2000/01 and 2008/09. In recessions economic data more frequently show negative surprises and lead to large forecasts revisions, reflected as large values of "news" as information measure. The variance of news is positively correlated with news, suggesting higher values of news are accompanied by more variation of news. Thus when there are more surprises in real time economic data, the information of different forecasters or how they treat this news varies more. It resembles the saying "Happy families are all alike; every unhappy family is unhappy in its own way".

Here "news" is quite similar in concept to "forecast revisions". In order to see their relationship, we plot news versus forecast revisions at the individual level for output forecasts in Figure 16. There is a strong positive correlation between news and absolute value of forecast revisions. Using a panel data fixed effect model to regress news on differences of moments at individual level, we find that "news" is significantly correlated with the difference (or say revision) in means, variances and skewness of the density forecasts, but not kurtosis.

Regression: news on differences of forecast moments (1 to 4 moments)

| | Estimate | p-Value |
|-------------|----------|---------|
| (Intercept) | 0.017 | 0.0000 |
| x1 | 0.14*** | 0.0000 |
| x2 | 0.01*** | 0.0052 |
| x3 | 0.03*** | 0.0000 |

x4 -0.01 0.1628

We also find a strong correlation between news and uncertainty.

9. Summary and conclusion

We apply continuous distributions to histogram survey data in SPF and obtain aggregate variance, average variance and disagreement, as well as aggregate entropy, average entropy and information measures. These two sets of measures are largely analogous to each other and can be used as uncertainty. Output forecast uncertainty has experienced a permanent fall in the 1980/90s. Inflation forecast uncertainty sees such a permanent fall from the 1990s. Thus, compared to other studies, our measures suggest a more permanent reduction in overall uncertainty since the great moderation. In recession quarters the disagreement measure tends to increase more than individual uncertainty. Employing VAR, we find evidence that uncertainty shocks have negative effects on the real economy, and so do tightening monetary policies. We find insignificant price puzzle in the inflation VAR framework. We also use information theory to capture the common shock in the density forecasts, and find that it accounts for the bulk of the individual uncertainty. We calculate information divergence at individual level to capture "news" and find "news" is countercyclical.

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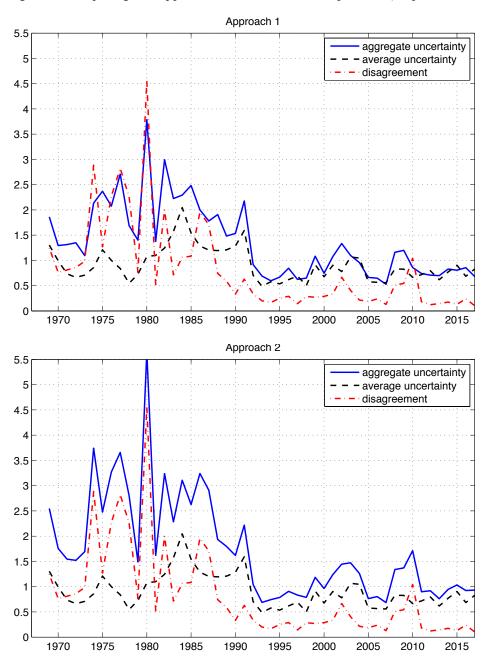
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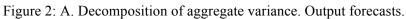
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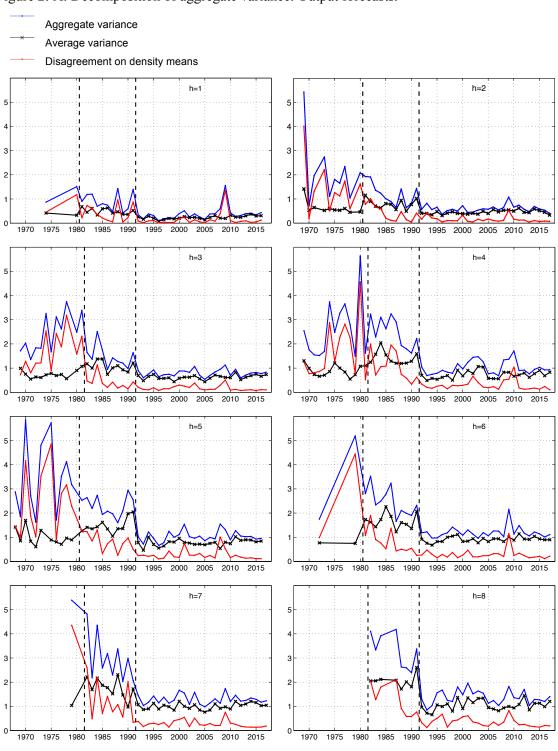
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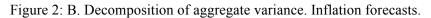
Figure 1: Comparing two approaches of variance decomposition (output forecasts, h=4)



Aggregate uncertainty: aggregate variance of the average distribution. Average uncertainty: average variance. Disagreement: disagreement on means. Approach 1: average distribution is the generalized beta distribution that is fitted to the average histogram. Approach 2: average distribution is the average of the fitted generalized beta distributions.







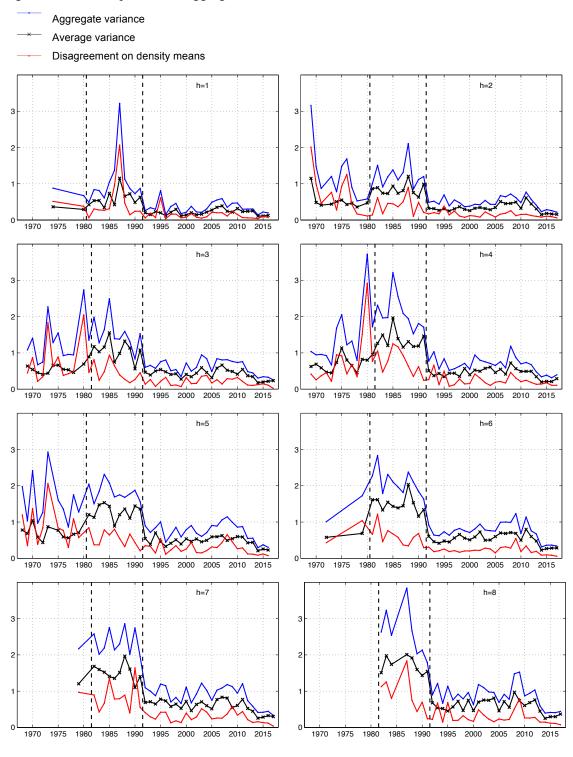


Figure 3: A. Entropy and information measure, output forecasts

Entropy of aggregate distribution

Average entropy

. Information measure

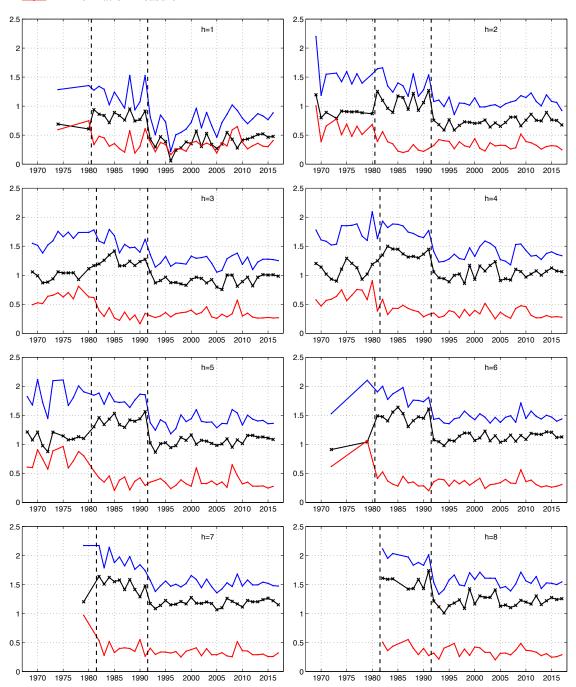


Figure 3: B. Entropy and information measure, inflation forecasts

Entropy of aggregate distribution

→ Average entropy

Information measure

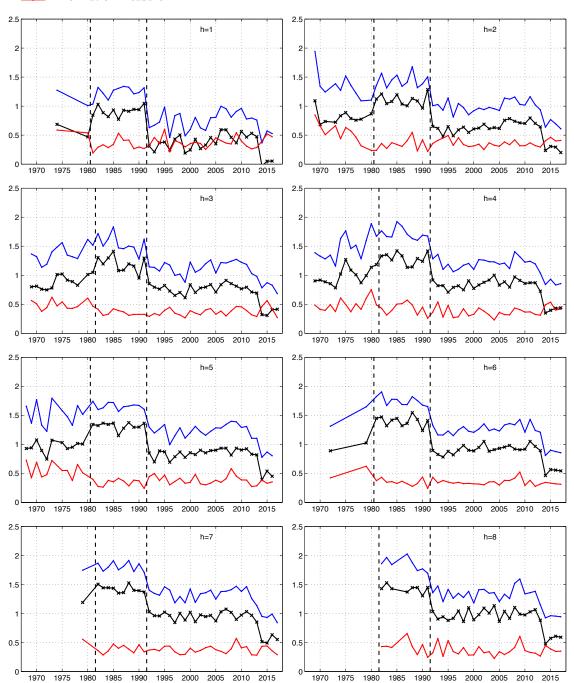
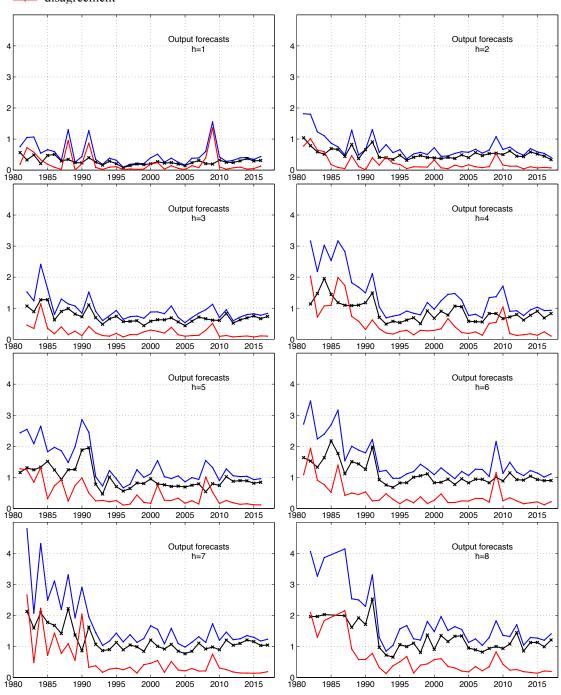


Figure 4: A. Variance and its components, corrected for bin-length. Output forecasts, 1981Q3 to 2017Q3.

- Variance of aggregate distribution
- → Average variance
- ... disagreement



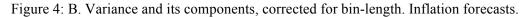


Figure 5: A. Entropy and its components, corrected for bin-length. Output forecasts, 1981Q3 to 2017Q3.

- --- Entropy of aggregate distribution
- __ Average entropy
 - Information measure

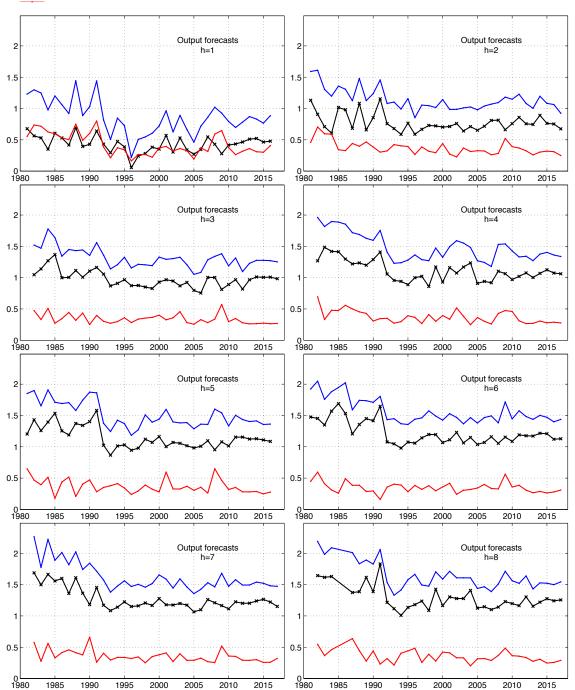


Figure 5: B. Entropy and its components, corrected for bin-length. Inflation forecasts.

Entropy of aggregate distribution

Average entropy

__ Information measure

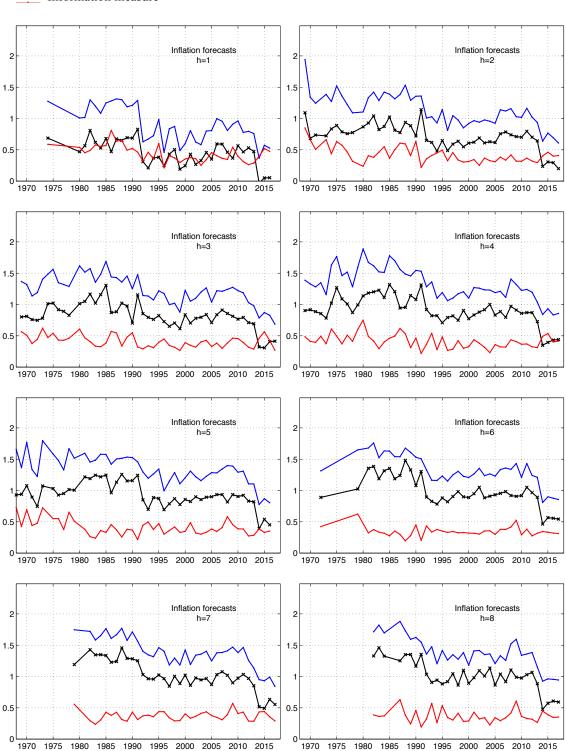
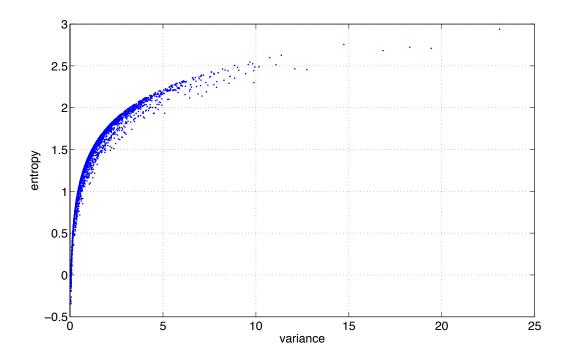


Figure 6: High correlation between Entropy and Variance

Entropy (Y) v.s. variance (X) (output forecasts)



Entropy (Y) v.s. log(variance) (X)

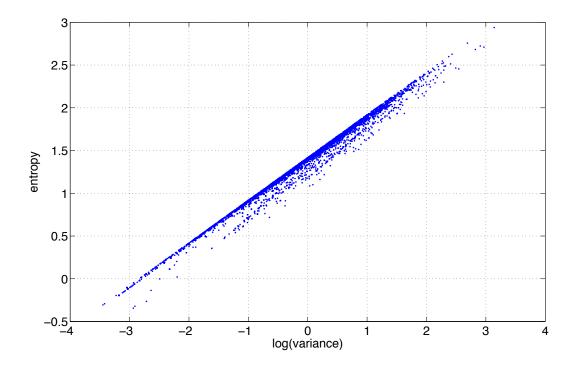
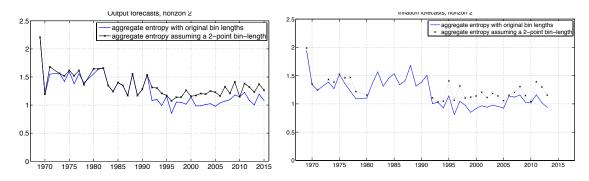


Figure 7. Effect of bin length on uncertainty, h=2

Entropy:



Variance:

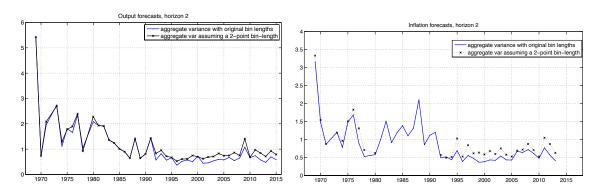
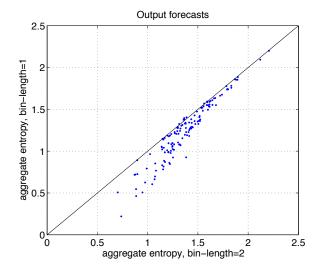


Figure 8. Effects of bin-length on uncertainty measures and the correction. Bin length=1 v.s. bin length=2



Seeing the correlation, we use the below regressions to convert 1981Q3-1991Q4 uncertainty measures (from surveys when bin length is 2) to their 1-point-bin-length equivalents.

Two regressions: Aggregate entropy=a+b*aggregate entropy assuming 2-point bin length 1. (entropy>1.5)

Estimated Coefficients:

| | Estimate | SE | ısıaı | p v arue |
|-------------|----------|-------|--------|----------|
| (Intercept) | -0.056 | 0.057 | -0.987 | 0.331 |
| x1 | 1.01 | 0.033 | 30.667 | 2.69e-25 |

Number of observations: 34, Error degrees of freedom: 32

Root Mean Squared Error: 0.0303

R-squared: 0.967, Adjusted R-Squared 0.966

F-statistic vs. constant model: 940, p-value = 2.69e-25

2. (entropy<1.5)

Estimated Coefficients:

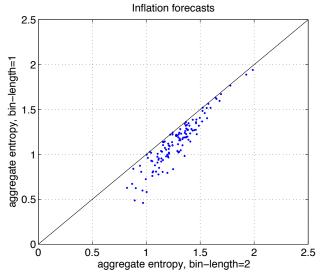
| | Estimate | SE | tStat | pValue |
|-------------|----------|-------|-------|----------|
| (Intercept) | -0.506 | 0.078 | -6.49 | 3.01e-09 |
| x1 | 1.276 | 0.061 | 20.84 | 6.42e-39 |

Number of observations: 106, Error degrees of freedom: 104

Root Mean Squared Error: 0.116

R-squared: 0.807, Adjusted R-Squared 0.805

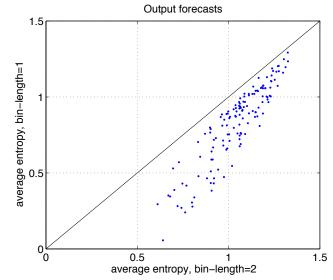
F-statistic vs. constant model: 397, p-value =6.42e-39



Regression:

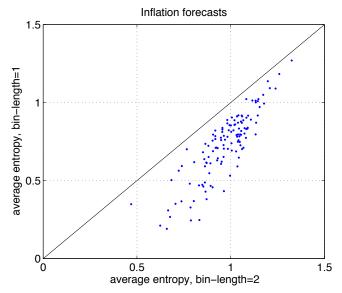
Aggregate entropy=a+b*aggregate entropy assuming 2-point bin length Y=0.976X+0.006

Average entropy:



Regression:

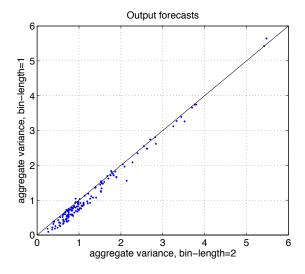
Average entropy=a+b*average entropy assuming 2-point bin length Y=1.446X-0.681



Regression:

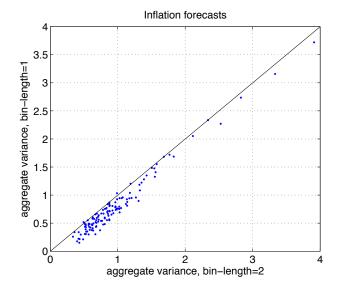
Average entropy=a+b*average entropy assuming 2-point bin length Y=1.307X-0.540

Aggregate variance:



Regression:

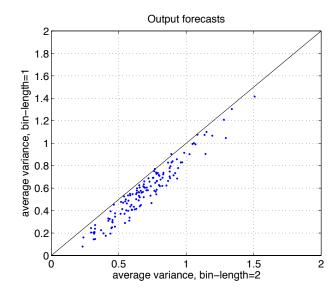
Aggregate variance=a+b*aggregate variance assuming 2-point bin length Y=1.030X-0.174



Regression:

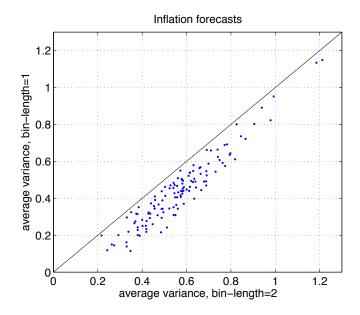
Aggregate variance=a+b*aggregate variance assuming 2-point bin length Y=1.003X-0.160

Average variance:



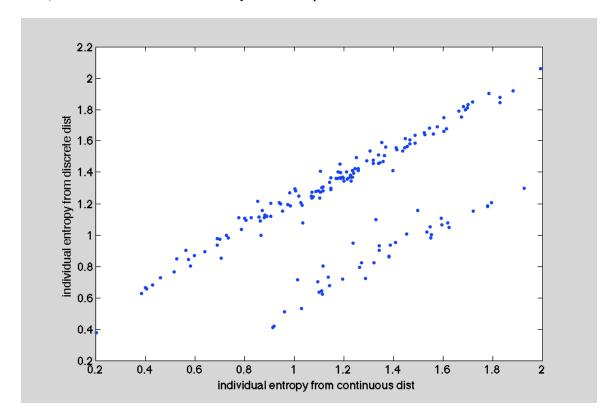
Regression:

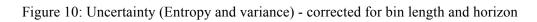
Average variance=a+b*average variance assuming 2-point bin length Y=1.020X-0.134



Regression: Average variance=a+b*average variance assuming 2-point bin length Y=1.016X-0.136

Figure 9: Compare with Shoja and Soofi (2017) (discrete data), fitting with continuous distributions is more reasonable than discrete data: there is a clear disconnection (two parallel lines) which reflects some discontiuity in the computation in the discrete case.





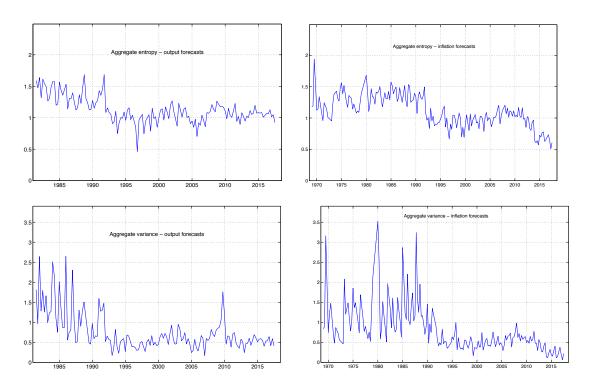
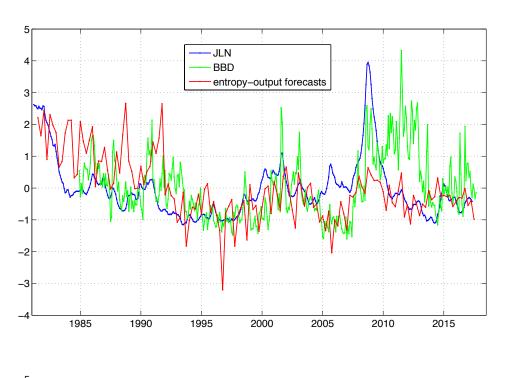


Figure 11: Uncertainty compared with other works



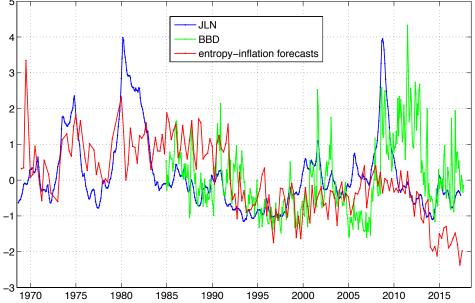
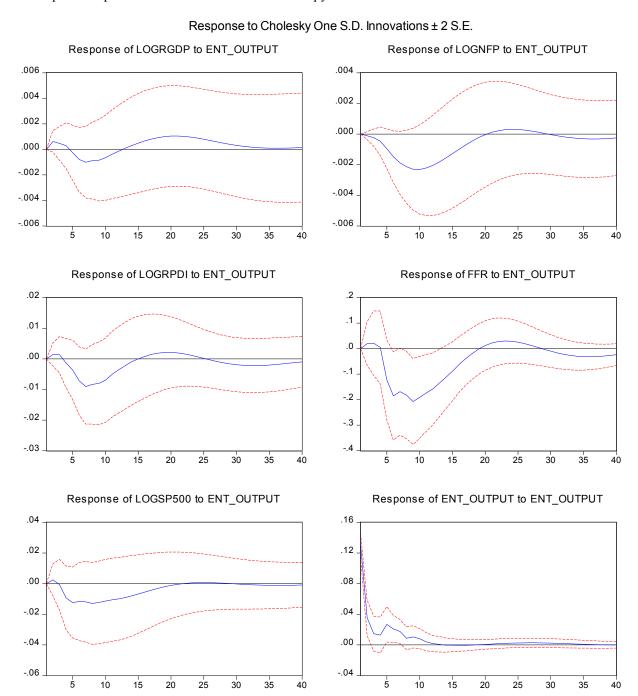


Figure 12: Impulse response functions of VAR models invloving output forecast uncertainty 6-variable VAR with four lags and cholesky ordering as following: log of real GDP, log of private domestic investment, log of nonfarm payroll, federal funds rate, log of S&P500 index and aggregate entropy of output forecasts. Sample: 1981Q3 to 2015Q4.

A. Impulse responses to a one S.D. shock of Entropy.



B. Impulse responses to a one S.D. shock of federal funds rate.

Response to Cholesky One S.D. Innovations ± 2 S.E.

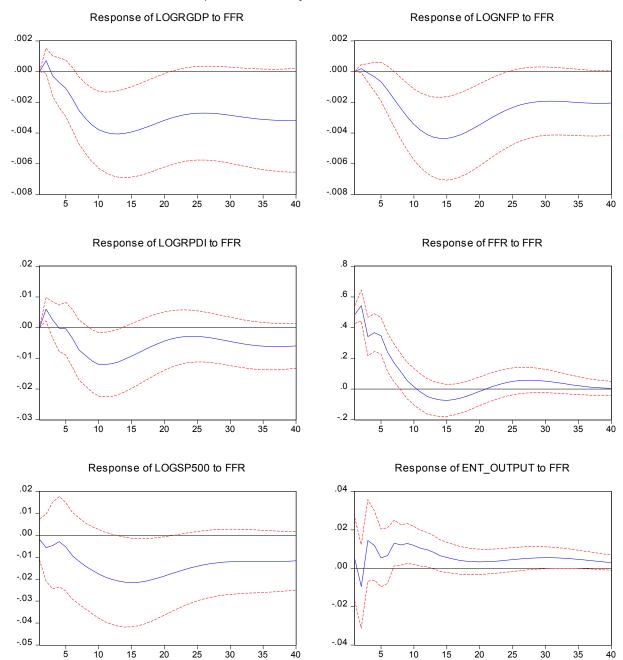
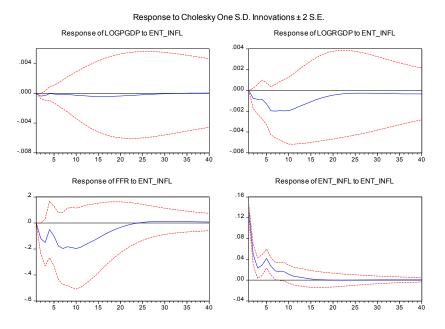
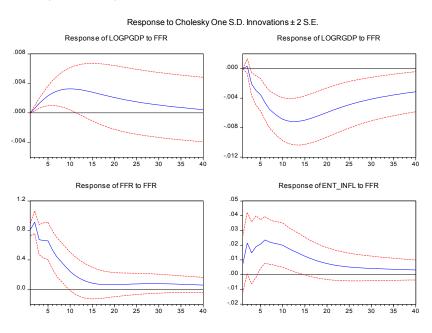


Figure 13: Impulse response functions of VAR models invloving inflation forecast uncertainty 4-variable VAR with four lags and cholesky ordering as following: log of price level (GDP deflator), log of real GDP, federal funds rate, and aggregate entropy of inflation forecasts.

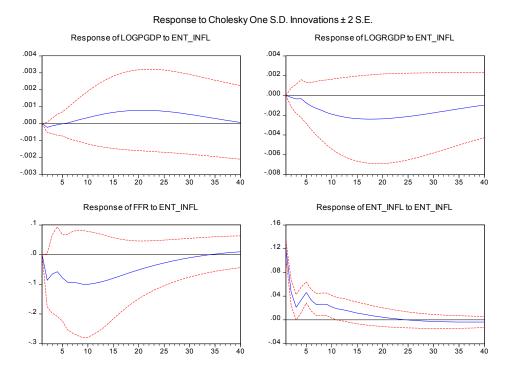
A. Impulse response functions to a one standard deviation increase of inflation uncertainty (entropy). Sample 1968Q4 to 2015Q4.



B. Impulse response functions to a one standard deviation increase of federal funds rate. Sample 1968Q4 to 2015Q4.



C. Impulse response functions to a one standard deviation increase of inflation uncertainty (entropy). Sample 1981Q3 to 2015Q4.



D. Impulse response functions to a one standard deviation increase of federal funds rate. Sample 1981Q3 to 2015Q4.

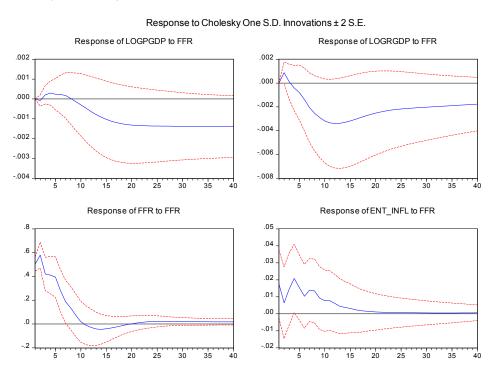
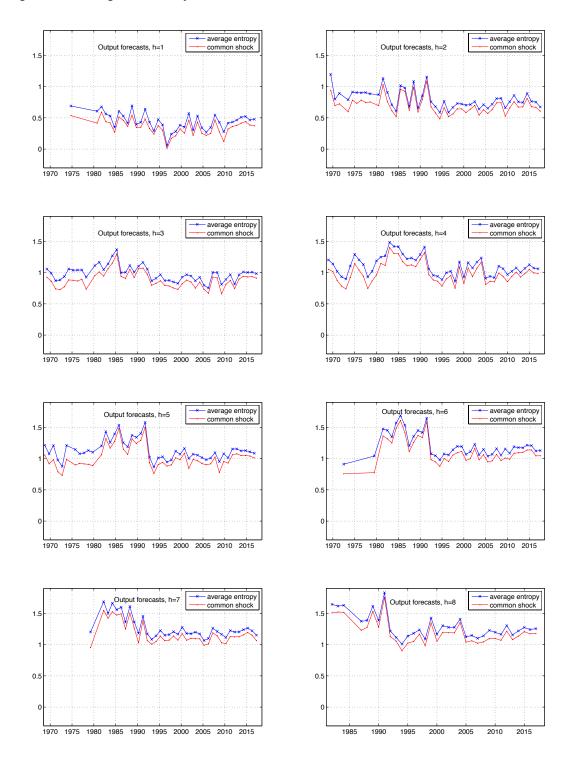


Figure 14: Average uncertainty and common shock



Inflation forecasts:

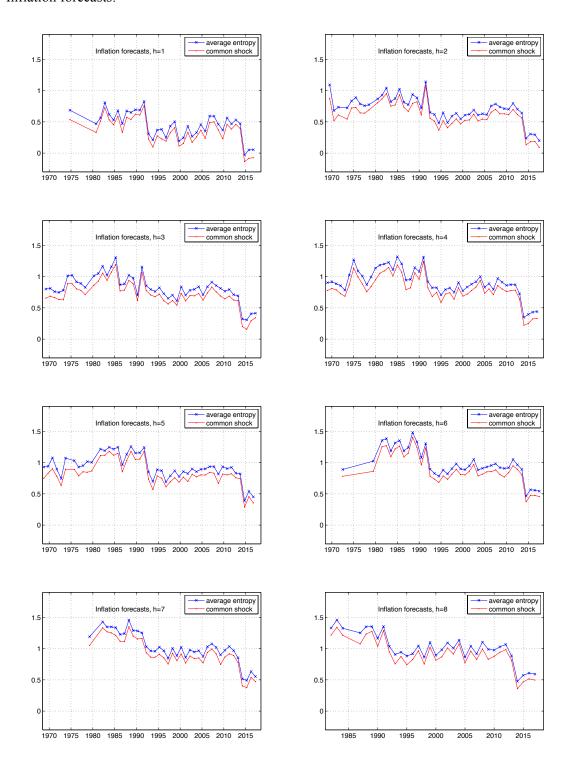
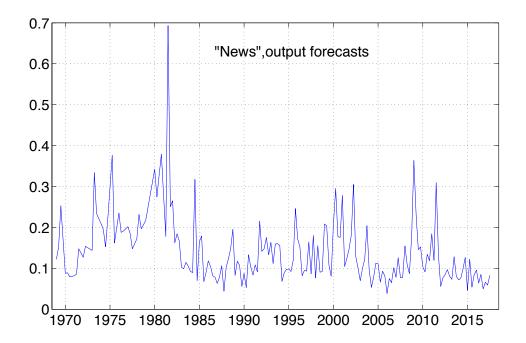


Figure 15: A. Measure of "News" using informatics

Output forecasts:



Inflation forecasts:

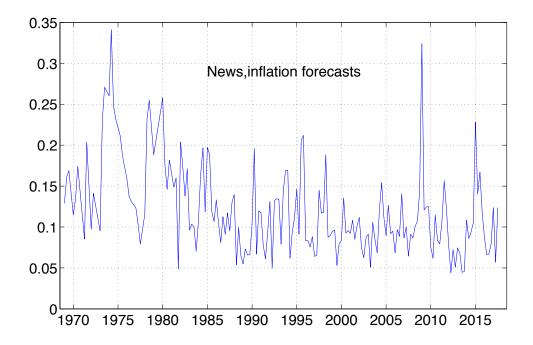
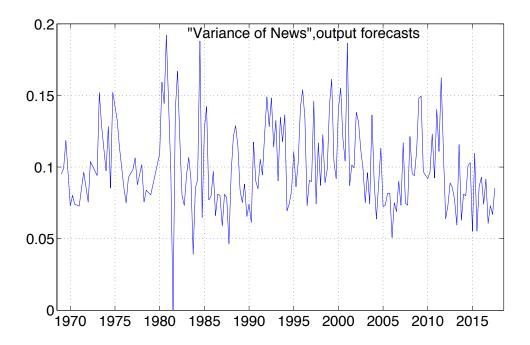


Figure 15: B. Variance of news

Output forecasts



Inflation forecasts:

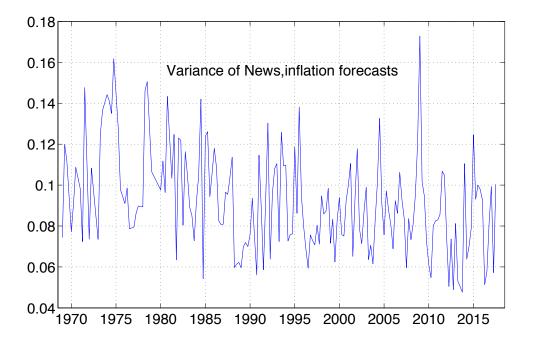
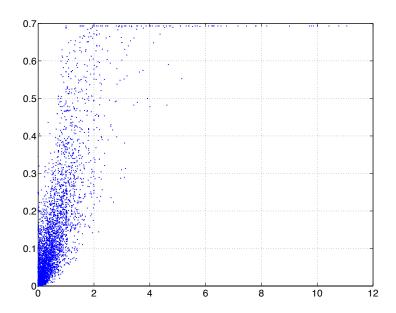


Figure 16: news v.s. forecast revisions.

X: forecast revisions of the forecast means

Y: news



news v.s. variance revisions

X: forecast revisions of the variances

Y: news

